Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

CAMBRIDGE International Examinations

[Turn over

1 Express $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

2 Find the coefficient of $\frac{1}{x}$ in the expansion of $\left(x - \frac{2}{x}\right)^5$. [3]

3 The common ratio of a geometric progression is 0.99. Express the sum of the first 100 terms as a percentage of the sum to infinity, giving your answer correct to 2 significant figures. [5]

4 A curve with equation $y = f(x)$ passes through the point $A\ (3,\ 1)$ and crosses the y-axis at B. It is given that $f'(x) = (3x - 1)^{-\frac{1}{3}}$. Find the y-coordinate of B. [6]

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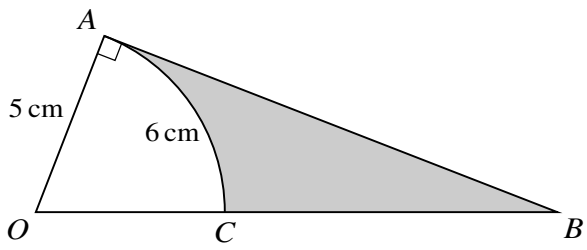

The diagram shows a triangle OAB in which angle $OAB = 90°$ and $OA = 5$ cm. The arc AC is part of a circle with centre O. The arc has length 6 cm and it meets OB at C. Find the area of the shaded region. [5]

6 The coordinates of points A and B are $(-3k - 1,\ k + 3)$ and $(k + 3,\ 3k + 5)$ respectively, where k is a constant $(k \neq -1)$.

(i) Find and simplify the gradient of AB, showing that it is independent of k. [2]

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(ii) Find and simplify the equation of the perpendicular bisector of AB. [5]

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7 **(a)** **(i)** Express $\dfrac{\tan^2\theta - 1}{\tan^2\theta + 1}$ in the form $a\sin^2\theta + b$, where a and b are constants to be found. [3]

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(ii) Hence, or otherwise, and showing all necessary working, solve the equation

$$\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = \frac{1}{4}$$

for $-90° \leqslant \theta \leqslant 0°$. [2]

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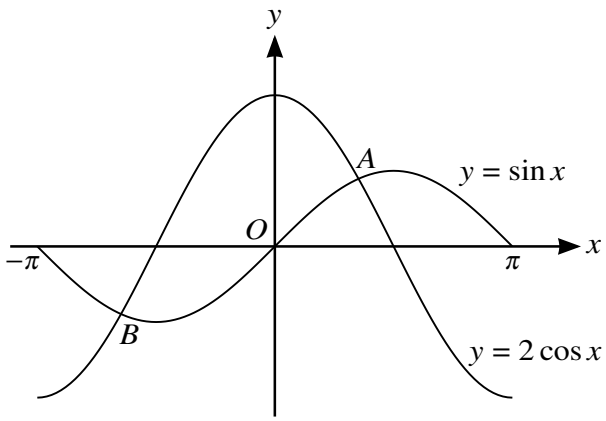
(b)



The diagram shows the graphs of $y = \sin x$ and $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$. The graphs intersect at the points A and B.

(i) Find the x-coordinate of A. [2]

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(ii) Find the y-coordinate of B. [2]

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8 **(i)** The tangent to the curve $y = x^3 - 9x^2 + 24x - 12$ at a point A is parallel to the line $y = 2 - 3x$. Find the equation of the tangent at A. [6]

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(ii) The function f is defined by $f(x) = x^3 - 9x^2 + 24x - 12$ for $x > k$, where k is a constant. Find the smallest value of k for f to be an increasing function. [2]

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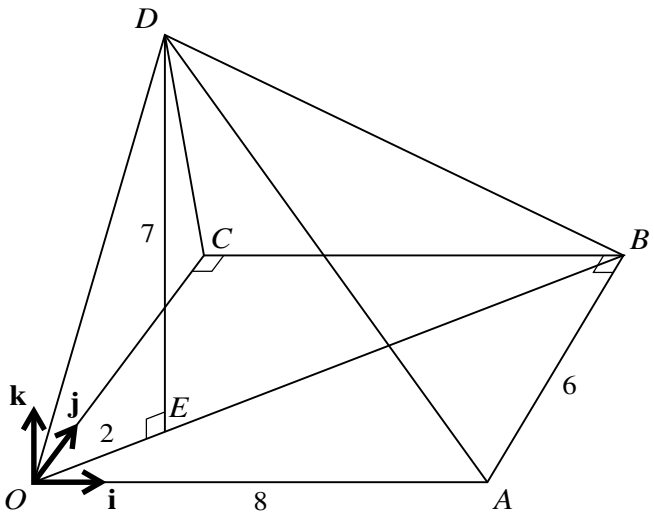

The diagram shows a pyramid $OABCD$ with a horizontal rectangular base $OABC$. The sides OA and AB have lengths of 8 units and 6 units respectively. The point E on OB is such that $OE = 2$ units. The point D of the pyramid is 7 units vertically above E. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and ED respectively.

(i) Show that $\overrightarrow{OE} = 1.6\mathbf{i} + 1.2\mathbf{j}$. [2]

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(ii) Use a scalar product to find angle BDO. [7]

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10 The one-one function f is defined by $f(x) = (x - 2)^2 + 2$ for $x \geqslant c$, where c is a constant.

(i) State the smallest possible value of c. [1]

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In parts **(ii)** and **(iii)** the value of c is 4.

(ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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(iii) Solve the equation $\mathrm{ff}(x) = 51$, giving your answer in the form $a + \sqrt{b}$. [3]

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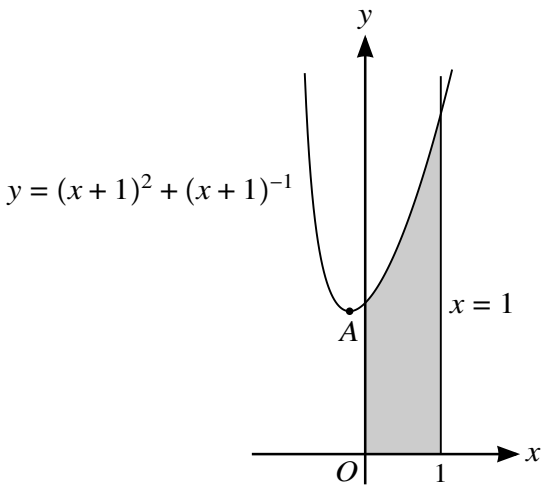

The diagram shows part of the curve $y = (x + 1)^2 + (x + 1)^{-1}$ and the line $x = 1$. The point A is the minimum point on the curve.

(i) Show that the x-coordinate of A satisfies the equation $2(x + 1)^3 = 1$ and find the exact value of $\frac{d^2y}{dx^2}$ at A. [5]

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(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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