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MATHEMATICS **9709/63**

Paper 6 **October/November 2017**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

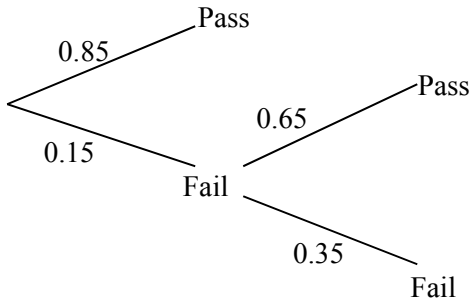
Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	*EITHER:* P(at least 1 completes) = 1 – P(0 people complete) $= 1 - (0.8)^3$	**(M1**	Fully correct unsimplified expression $1 - (0.8)^3$ OE
	$= 0.488\left(\frac{61}{125}\right)$	**A1)**	
	OR1: $P(1, 2, 3) = {}^3C_1(0.2)(0.8)^2 + {}^3C_2(0.2)^2(0.8) + (0.2)^3$	**(M1**	Unsimplified correct 3 term expression
	$= 0.488\left(\frac{61}{125}\right)$	**A1)**	
	OR2: $0.2 + 0.8 \times 0.2 + 0.8 \times 0.8 \times 0.2$	**(M1**	Unsimplified sum of 3 correct terms
	$= 0.488\left(\frac{61}{125}\right)$	**A1)**	
		2	

Question	Answer	Marks	Guidance
2	$\Sigma(x-45) = 1218 - 20 \times 45 = 318$	**B1**	
	$\frac{\Sigma(x-45)^2}{20} - \left(\frac{\Sigma(x-45)}{20}\right)^2 = 4.2^2$	**M1**	Fully correct substitution in the correct coded variance formula with their $\Sigma(x-45)$ **OR** valid method for $\Sigma x^2 = 74\,529$ ($4.2^2 = \frac{\Sigma x^2}{20} - \left(\frac{1218}{20}\right)^2$) and expanding $\Sigma(x-45)^2$ correctly $= \Sigma x^2 - 90\Sigma x + 20 \times 45^2 = '74\,529' - 90 \times 1218 + 40\,500 = 5409$
	$\Sigma(x-45)^2 = 5409$	**A1**	
		3	

Question	Answer	Marks	Guidance
3(i)		**M1**	Correct shape
		A1	All correct labels and probabilities
		2	

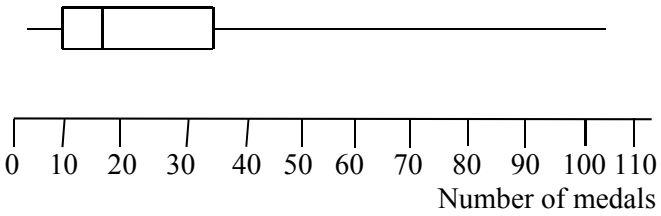
Question	Answer	Marks	Guidance
3(ii)	$\mathrm{P}(F \mid P) = \dfrac{\mathrm{P}(F \cap P)}{\mathrm{P}(P)}$	**M1**	P(P) consistent with their tree diagram seen anywhere
	$= \dfrac{0.15 \times 0.65}{0.85 + 0.15 \times 0.65}$ or $\dfrac{0.15 \times 0.65}{1 - 0.15 \times 0.35}$	**A1**	Correct unsimplified P(P) seen as num or denom of a fraction
	$= \dfrac{0.0975}{0.9475}$	**M1**	P($F \cap P$) found as correct product or consistent with their tree diagram seen as num or denom of a fraction
	$= \dfrac{39}{379} = 0.103$	**A1**	
		4	

Question	Answer	Marks	Guidance
4(i)		**B1**	At least 3 different correct values of X (can be unsimplified)
		B1	Four correct probabilities in a Probability Distribution table
		B1	Correct probs with correct values of X
		3	

x	–3	0	5	32
Prob	1/6	1/2	1/6	1/6

Question	Answer	Marks	Guidance
4(ii)	$E(X) = -3/6 + 5/6 + 32/6 = 34/6 = 17/3$ (5.67)	**M1**	Subst their attempts at scores in correct formula as long as 'probs' sum to 1
	$Var(X) = 9/6 + 25/6 + 1024/6 - (34/6)^2$	**M1**	Subst their attempts at scores in correct var formula
	$= 144\left(\frac{1298}{9}\right)$	**A1**	Both answers correct
		3	

Question	Answer	Marks	Guidance
5(i)	0 | 2 2 5 6 9 1 | 0 0 0 2 2 3 3 4 7 7 8 8 2 | 8 8 3 | 4 5 8 4 | 4 5 | 6 | 5 7 | 8 | 2 8 9 | 10 | 4 key 2 | 8 means 28 medals	**B1**	Stem, digits 5, 7, 9 can be missing here, can be upside down
		B1	All leaves in correct order increasing from stem, (5, 7 and 9 can be missing), condone commas
		B1	Reasonable shape, requires all values of the stem, only one line for each stem and leaves must be lined up. Can be upside down or sideways. No commas. Condone one 'leaf' error.
		B1	Correct key must state 'medals' or have 'medals' in leaf heading or title
		4	

Question	Answer	Marks	Guidance
5(ii)	Med = 17 LQ = 10 UQ = 35  	**B1**	Median correct
		B1	LQ and UQ correct
		B1	Uniform scale from 2 to 104 (need 3 identified points min) and label including medals (can be in title)
		B1 FT	Correct box med and quartiles on diagram, FT their values
		B1	Correct end-whiskers from ends of box but not through box
		5	

Question	Answer	Marks	Guidance
6(i)	${}^{18}P_5$	**M1**	${}^{18}P_x$ or ${}^{y}P_5$ OE seen, $0 < x < 18$ and $5 < y < 18$, can be mult by $k \geqslant 1$
	= 1 028 160	**A1**	
		2	

Question	Answer	Marks	Guidance
6(ii)	*EITHER:* e.g. ***(CCCCC)********** in 5!×14 ways	**(B1**	5! OE mult by k ⩾ 1, considering the arrangements of cars next to each other
	= 1680	**B1**	Mult by 14 OE, (or 14 on its own) considering positions within the line
	P (next to each other) = 1680/1 028 160	**M1**	Dividing by **(i)** for probability
	P(not next to each other) = 1 – 1680/1 028 160	**M1**	Subtracting prob from 1 (or their ' 5! × 14 ' from **(i)**)
	$= 0.998\left(\frac{611}{612}\right)$ OE	**A1)**	
	OR1: $\frac{5! \times 14!}{18!} = 0.001634$	**(B1**	5! OE mult by k ⩾ 1 (on its own or in numerator of fraction) considering the arrangements of cars next to each other
		B1	Multiply by 14!, (or 14! on its own) considering all ways of arranging spaces with 5 cars together
		M1	Dividing by 18!, total number of ways of arranging spaces
	1 – 0.001634	**M1**	Subtracting prob from 1 (or '5! × 14!' from 18!)
	= 0.998(366)	**A1)**	
	OR2: 4 together – $2\times5!\times14C12 = 21\,840$ 3, 1, 1 – $3\times5!\times14C11 = 131\,040$ 3, 2 – $2\times5!\times14C12 = 21\,840$ 2,2,1 – $3\times5!\times14C11 = 131\,040$ 2,1,1,1 – $4\times5!\times14C10 = 480\,480$ 1,1,1,1,1 – $5!\times14C9 \text{ or } 14P5 = 240\,240$	**(M1**	Listing the six correct scenarios (only): 4 together; 3 together and 2 separate; 3 together and 2 together; two sets of 2 together and 1 separate; 2 together and 3 separate; 5 separate.
		M1	Summing total of the six scenarios, at least 2 correct unsimplified

Question	Answer	Marks	Guidance
	Total = 1 026 480	**A1**	Total of 1 026 480
		M1	Dividing their 1 026 480 by their **6(i)**
	$1\,026\,480 \div 1\,028\,160 = 0.998(366)$	**A1)**	
		5	

Question	Answer	Marks	Guidance
6(iii)	R(5) W(4) B(3) Scenarios No. of ways 1 1 1 $= 5 \times 4 \times 3 = 60$ 0 1 2 $= 4 \times {}^3C_2 = 12$ 0 2 1 $= {}^4C_2 \times 3 = 18$ 1 0 2 $= 5 \times {}^3C_2 = 15$ 2 0 1 $= {}^5C_2 \times 3 = 30$ 1 2 0 $= 5 \times {}^4C_2 = 30$ 2 1 0 $= {}^5C_2 \times 4 = 40$	**B1**	$5C1 \times 4C1 \times 3C1$ or better seen i.e. no. of ways with 3 different colours
		M1	Any of ${}^5C_2$ or ${}^4C_2$ or ${}^3C_2$ seen multiplied by $k > 1$ (can be implied)
		A1	2 correct unsimplified 'no. of ways' other than 5C1 × 4C1 × 3C1
		M1	Summing no more than 7 scenario totals containing at least 6 correct scenarios
	Total = 205	**A1**	
	OR		
	${}^{12}C_3 –$	**M1**	Seeing '${}^{12}C_3 –$', considering all selections of 3 cars
	$– {}^5C_3$	**M1**	Subt ${}^5C_3$ OE, removing only red selections
	$– {}^4C_3$	**M1**	Subt ${}^4C_3$ OE, removing only white selections
	$– {}^3C_3$	**M1**	Subt ${}^3C_3$ OE, removing only black selections
	$= 205$	**A1**	Correct answer
		5	

Question	Answer	Marks	Guidance
7(i)	$P(t > 6) = P\left(z > \frac{6-5.3}{2.1}\right) = P(z > 0.333)$	**M1**	Standardising, no continuity correction, no sq, no sq rt
	= 1 – 0.6304	**M1**	Correct area 1 – Φ (< 0.5), final solution
	= 0.370 or 0.369	**A1**	
		3	
7(ii)	$z = 1.645$	**B1**	± 1.645
	$1.645 = \frac{x-5.3}{2.1}$	**M1**	Standardising, no continuity correction, allow sq, sq rt. Must be equated to a z-value
	$x = 8.75$ or 8.755 or 8.7545	**A1**	
		3	
7(iii)	$n = 10$, $p = 0.05$	**M1**	Bin term ${}^{10}C_x\, p^x(1-p)^{10-x}$
	$P(0, 1, 2) = (0.95)^{10} + {}^{10}C_1(0.05)(0.95)^9 + {}^{10}C_2(0.05)^2(0.95)^8$	**M1**	Correct unsimplified answer
	= 0.988 (0.9885 to 4 sf)	**A1**	
		3	
7(iv)	P(misses bus) = $P(t < 0)$	***M1**	Seeing t linked to zero
	$= P\left(z < \frac{0-5.3}{2.1}\right) = P(z < -2.524) = 1 - \Phi(2.524)$ = 1 – 0.9942	**DM1**	Standardising with $t = 0$, no continuity correction, no sq, no sq rt
	= 0.0058	**A1**	
		3	