

Cambridge
International
AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS

9709/61

Paper 6

May/June 2017

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This document consists of **12** printed pages.

Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

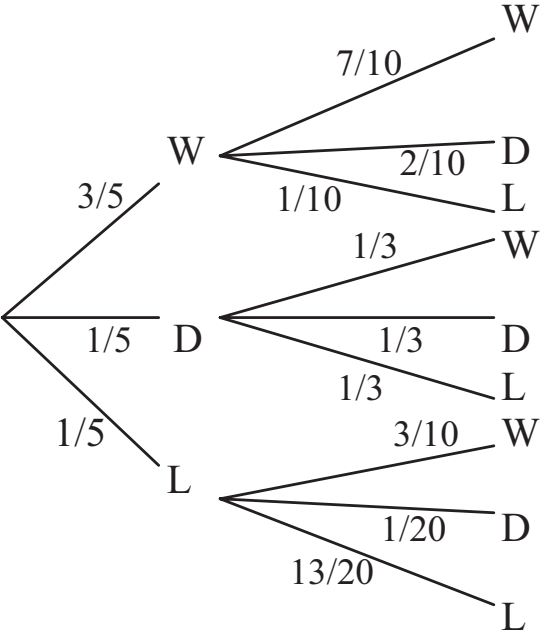
Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

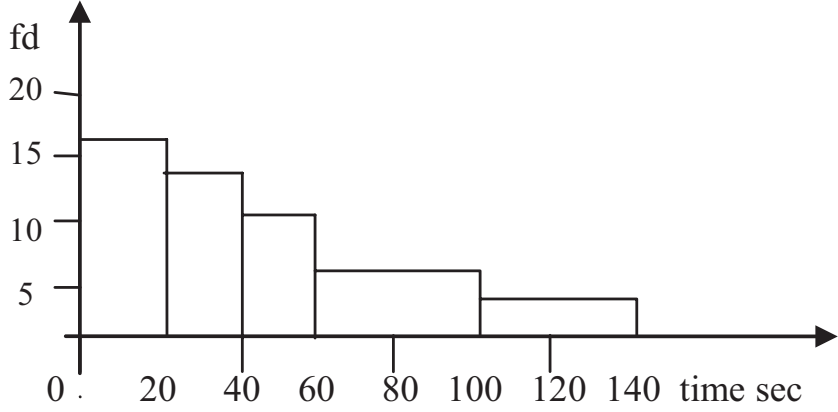
PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	<i>EITHER:</i> $\frac{\sum x}{30} - k = \frac{315}{30} = 10.5$	(M1)	Dividing 315 by ± 30 and + or – from 50.5 need both and no more
	$k = 5.5 - 10.5 = 40$	A1)	Correct answer from correct working
	<i>OR:</i> $\sum x = 50.5 \times 30 = 1515, 1515 - 30k = 315$	(M1)	Mult by 50.5 by 30 and + or – 315 and dividing by ± 30 need all these
	$k = 40$	A1)	Correct answer from correct working. 1200 gets M0
	Total:	2	
1(ii)	<i>EITHER:</i> $\text{var} = 4022/30 - 10.5^2 (=23.817)$	(M1)	Subst in correct coded variance formula
	$\text{sd} = 4.88$	A1)	
	<i>OR:</i> $\sum x^2 - 2(40)\sum x + 30(40)^2 = 4022, \sum x^2 = 77222$ $\text{Var} = 77222/30 - 50.5^2 (= 23.817)$	(M1)	Expanding with $\pm 40\sum x$ and $\pm 30(40)^2$ seen
	$\text{sd} = 4.88$	A1)	
	Total:	2	

Question	Answer	Marks	Guidance
2	$P(R) = 4/36 = 1/9$	M1	Attempt at $P(R)$ by probability space diag or listing more than half the options, must see a prob, just a list is not enough
	$P(T) = P(O, E) + P(E, O) = 1/4 + 1/4 = 1/2$ OR $P(R T) = 1/9$	M1	Attempt at $P(T)$ or $P(R T)$ involving more than half the options
	$P(R \cap T) = P(3, 4) + P(4, 3) = 2/36 = 1/18$ OR $P(R T) = 1/9$	B1	Value stated, not from $P(R) \times P(T)$ e.g. from probability space diagram
	As $P(R) \times P(T) = P(R \cap T)$ OR as $P(R T) = P(R)$	M1	Comparing product values with $P(R \cap T)$, or comparing $P(R T)$ with $P(R)$
	The events are independent.	A1	Correct conclusion must have all probs correct
	Total:		5

Question	Answer	Marks	Guidance
3(i)		<p>M1</p>	<p>Correct shape i.e. 3 branches then 3 by 3 branches, labelled and clear annotation Condone omission of lines for first match result providing the probabilities are there.</p>
		<p>A1</p>	<p>All correct probs with fully correct shape and probs either fractions or decimals not 1.5/5 etc.</p>
	<p>Total:</p>	<p>2</p>	

Question	Answer	Marks	Guidance
3(ii)	$P(L_1 \text{ given } W_2) = \frac{P(L_1 \cap W_2)}{P(W_2)}$	M1	Attempt at $P(L_1 \cap W_2)$ as a two-factor prod only as num or denom of a fraction
	$= \frac{1/5 \times 3/10}{3/5 \times 7/10 + 1/5 \times 1/3 + 1/5 \times 3/10}$	M1	Attempt at $P(W_2)$ as sum of appropriate 3 two-factor probs OE seen anywhere
		A1	Unsimplified correct $P(W_2)$ num or denom of a fraction
	$= \frac{3/50}{41/75} = 9/82(0.110)$	A1	
	Total:	4	

Question	Answer	Marks	Guidance
4(i)	fd 16, 14, 11, 505, 2.5	M1	Attempt at fd (must be at least 3 freq/cw) – may be implied by graph
		A1	Correct heights seen on graph i.e. must see a gap for fd = 2.5 etc.
		B1	Correct end points of bars and correct widths
		B1	labels fd, sec. Time can be optional. Linear axes, condone $0 \leq t < 20$ etc.
	Total:	4	

Question	Answer	Marks	Guidance
4(ii)	$(10 \times 320 + 30 \times 280 + 50 \times 220 + 80 \times 220 + 120 \times 100) / 1140$	M1	using $\Sigma fx / n$ with mid-point attempt ± 0.5 , not ends not class widths
	= 45.8	A1	
	Total:	2	
5(i)	$p = 0.07$	B1	
	$P(2) = {}^{20}C_2 (0.07)^2 (0.93)^{18}$	M1	Bin term ${}^{20}C_x p^x (1-p)^{20-x}$ their p
	= 0.252	A1	
	Total:	3	
5(ii)	$P(\text{at least 1 cracked egg}) = 1 - (0.93)^{20} = 1 - 0.2342$	M1	Attempt to find $P(\text{at least 1 cracked egg})$ with their p from (i) allow $1 - P(0, 1)$ OE
	= 0.766	A1	Rounding to 0.766
	Total:	2	
5(iii)	$(0.7658)^n < 0.01$	M1	Eqn or inequal containing (their 0.766) ⁿ or (their 0.234) ⁿ , together with 0.01 or 0.99
	$n = 18$	A1	
	Total:	2	

Question	Answer	Marks	Guidance
6(a)(i)	$z = 0.674$	B1	rounding to ± 0.674 or 0.675
	$0.674 = \frac{6.8 - \mu}{0.25\mu}$	M1	standardising, no cc, no sq rt, no sq, σ may still be present on RHS
		M1	subst and sensible solving for μ must collect terms, no z -value needed can be 0.75 or 0.7734 need a value for μ
	$\mu = 5.82$	A1	
	Total:	4	
6(a)(ii)	$P(X < 4.7) = P\left(z < \frac{4.7 - 5.819}{1.4548}\right)$	M1	\pm standardising no cc, no sq rt, no sq unless penalised in (a)(i)
	$= \Phi(-0.769) = 1 - 0.7791$	M1	correct side for their mean i.e. $1 - \Phi$ (final solution)
	$= 0.221$	A1	
	Total:	3	
6(b)	$P(< 15.75) = P\left(z < \frac{15.75 - 16}{0.2}\right) = 1 - P(z < 1.25) = 1 - 0.8944 = 0.1056$ and $P(> 16.25) = 0.1056$ by sym	*M1	Standardising for 15.75 or 16.25 no cc no sq no sq rt unless penalised in (a)(i) or (a)(ii)
	$P(\text{usable}) = 1 - 0.2112 = 0.7888$	B1	$2\Phi - 1$ OE for required prob, (final solution)
	Usable rods = $1000 \times 0.7888 =$	DM1	Mult their prob by 1000 dep on recognisable attempt to standardise
	788 or 789	A1	
	Total:	4	

Question	Answer	Marks	Guidance												
7(a)	<i>EITHER:</i> e.g. xxxxx =5! for the other children	(B1)	5! OE seen alone or mult by integer $k \geq 1$, no addition												
	Put y in 6 ways, then 5 then 4 for the youngest children	B1	Mult by 6P3 OE												
	Answer $5! \times 6P3 = 14400$	(B1)	Correct answer												
	<i>OR:</i> total – 3 tog – 2 tog = $8! - 6!3! - 6! \times 2 \times 5 \times 3 = 14400$	(B1)	$8! - 6! \times k \geq 1$ seen												
		B1	$6!3!$ or $6! \times 2 \times 5 \times 3$ seen subtracted												
		(B1)	Correct answer												
	Total:		3												
7(b)	<table style="border: none; width: 100%;"> <tr> <td style="text-align: center;">D</td> <td style="text-align: center;">W</td> <td style="text-align: center;">M</td> <td></td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">2</td> <td style="text-align: center;">2</td> <td style="text-align: center;">1</td> <td style="text-align: center;">=</td> <td style="text-align: center;">$6C2 \times 4C2 \times 1$</td> <td style="text-align: center;">= 90</td> </tr> </table>	D	W	M				2	2	1	=	$6C2 \times 4C2 \times 1$	= 90	B1	One correct unsimplified option
	D	W	M												
	2	2	1	=	$6C2 \times 4C2 \times 1$	= 90									
	<table style="border: none; width: 100%;"> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">1</td> <td style="text-align: center;">1</td> <td style="text-align: center;">=</td> <td style="text-align: center;">$6C3 \times 4 \times 1$</td> <td style="text-align: center;">= 80</td> </tr> </table>	3	1	1	=	$6C3 \times 4 \times 1$	= 80	M1	Summing 2 or more 3-factor options which can contain perms or 3 factors added. The 1 can be implied						
	3	1	1	=	$6C3 \times 4 \times 1$	= 80									
<table style="border: none; width: 100%;"> <tr> <td style="text-align: center;">1</td> <td style="text-align: center;">3</td> <td style="text-align: center;">1</td> <td style="text-align: center;">=</td> <td style="text-align: center;">$6 \times 4C3 \times 1$</td> <td style="text-align: center;">= 24</td> </tr> </table>	1	3	1	=	$6 \times 4C3 \times 1$	= 24	M1	Summing the correct 3 unsimplified outcomes only							
1	3	1	=	$6 \times 4C3 \times 1$	= 24										
Total=194 ways	A1														
Total:		4													

Question	Answer	Marks	Guidance
7(c)	$\begin{array}{cccc} \text{C} & \text{D} & \text{S} & \\ 2 & 1 & 1 & = \end{array} \quad {}^{26}\text{C}_2 \times 9 \times 5 \times 4! = 351\,000$	M1	summing 2 or more options of the form (2 1 1), (1 2 1), (1 1 2), can have perms, can be added
	$1 \quad 2 \quad 1 \quad = \quad 26 \times {}^9\text{C}_2 \times 5 \times 4! = 112\,320$	M1	4 relevant products seen excluding 4! e.g. $26 \times 9 \times 8 \times 5$ or $26 \times {}^9\text{P}_2 \times 5$ for 2nd outcome, condone $26 \times 9 \times 5 \times 37$ as being relevant
	$1 \quad 1 \quad 2 \quad = \quad 26 \times 9 \times {}^5\text{C}_2 \times 4! = 56\,160$	M1	mult all terms by 4! or 4!/2!
	Total = 519 480	A1	
	Total:	4	