CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the March 2016 series

9709 MATHEMATICS

9709/42 Paper 4 (Mechanics), maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the March 2016 series for most Cambridge IGCSE® and Cambridge International A and AS Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	KE gain = ½ × 105 × ($10^2 – 5^2$) WD against Resistance = 50 × 40 Total WD = 5937.5 J	M1 A1 B1	 3	Attempt KE gain **or** WD against Res Both correct (unsimplified) KE gain = 3937.5 J WD = 2000 J WD = KE gain + WD against Res
Alternative method				
	$10^2 = 5^2 + 2 \times 50 \times a$ [a = 0.75] DF – 40 = 105a DF = 40 + 105 × 0.75 = 118.75 Total WD = 118.75 × 50 = 5937.5 J	 M1 A1 B1	 3	Using $v^2 = u^2 + 2as$ and applying Newton's 2nd law to the system WD = DF × 50
2 (i)	DF = 1350 P = 1350 × 32 = 43.2 kW	B1 B1	 2	
(ii)	DF – 1350 – 1200g × 0.1 = 0 [DF = 2550] DF = 76500/v $v = 30\,\text{ms}^{-1}$	 M1 M1 A1	 3	For using Newton's 2nd law applied to the car up the hill (3 terms) Allow use of θ = 5.7° For using DF = P/v
3 (i)	 R_x = 40 × (24/25) – 30 × (7/25) [= 30] R_y = 50 – 40 × (7/25) – 30 × (24/25) [= 10] $R = \sqrt{R_x^2 + R_y^2}$ **and** $\theta = \tan^{-1}\left(R_y / R_x\right)$ R = 31.6 N **and** θ = 18.4° with the positive x-axis	M1 A1 M1 A1 M1 A1	 6	For resolving forces horizontally Allow R_x = 40 cos 16.3 – 30 sin 16.3 For resolving forces vertically Allow R_y = 50 – 40 sin16.3 – 30 cos16.3 For using Pythagoras to find the resultant force R **and** trigonometry to find the angle θ made by the resultant with the x-axis

Alternative method for 3(i)				
(i)		M1		Resolve forces along 40 N direction
	$R_1 = 40 – 50 \times (7/25)$ [= 26]	A1		Allow $R_1 = 40 – 50 \sin 16.3$
		M1		Resolve forces along 30 N direction
	$R_2 = 30 – 50 \times (24/25)$ [= –18]	A1		Allow $R_2 = 30 – 50 \cos 16.3$
	$R^2 = R_1^2 + R_2^2$ **and** $\arctan(–R_2/R_1)$	M1		Use Pythagoras **and** trigonometry
	$R = 31.6$ N **and** direction is $34.7 – \alpha = 18.4°$ with positive x–axis	A1	6	Using $\arctan(18/26) = 34.7°$ is the angle between R and the 40 N force
(ii)	$P = 40$	B1	1	
4 (i)	$5\cos\alpha = F$ [$F = 4$]	M1		For resolving forces horizontally Allow use of $\alpha = 36.9°$ throughout
	$R + 5\sin\alpha = 8$ [$R = 5$]	M1		For resolving forces vertically
	$4 = 5\mu$	M1		For using $F = \mu R$
	$\mu = 0.8$	A1	4	
(ii)	$R + 10\sin\alpha = 8$ [$R = 2$] **and** $F = 0.8 \times R$ [$F = 1.6$]	B1		For resolving forces vertically to find the new value of R **and** using $F = \mu R$
	$10\cos\alpha – F = 0.8a$	M1		For resolving horizontally
	$a = 8\text{ ms}^{-2}$	A1	3	
5 (i)	[$2500 – 2000g \times 0.1 – 250 = 2000a$]	M1		For using Newton's 2nd law for the system or for applying Newton's 2nd law to the car and to the trailer and for solving for a Allow use of $\alpha = 5.7°$ throughout
	$a = 1/8 = 0.125\text{ ms}^{-2}$	A1		
	$2500 – T – 100 – 1200g \times 0.1 = 1200 \times 0.125$ or $T – 150 – 800g \times 0.1 = 800 \times 0.125$	M1		For applying Newton's 2nd law either to the car or to the trailer to set up an equation for T
	$T = 1050$ N	A1	4	

(ii)	$-2000g \times 0.1 - 250 = 2000a$ $[a = -1.125]$	M1		For applying Newton's 2nd law to the system with no driving force to set up an equation for a
	$0 = 30 - 1.125t$	M1		For using $v = u + at$
	$t = 26.7$ s	A1	3	Allow $t = 80/3$ s
Alternative method for 5(ii)				
(ii)	[½ (2000) 30^2 = $250s + 2000 \times g \times 0.1s$] $\rightarrow s = 400$	M1		Apply work/energy equation to find s the distance travelled up the plane with no driving force (3 terms) as: KE loss = WD against F + PE gain
	$[400 = ½ (30 + 0)t]$	M1		For using $x = ½(u + v)t$
	$t = 26.7$ s	A1	3	Allow $t = 80/3$ s
6 (i)	$[T = 0.8a$ for A $2 - T = 0.2a$ for B $0.2g = (0.2 + 0.8)a$ system]	M1		For applying Newton's 2nd law either to particle A or to particle B or to the system
		M1		For applying N2 to a second particle (if needed) and solving for a
	$[a = 2]$	A1		
	$[2.5 = ½ \times 2 \times t^2]$	M1		A complete method for finding t such as using $s = ut + ½at^2$
	$t = 1.58$ s	A1	5	Allow $t = \frac{1}{2}\sqrt{10}$
First Alternative Method for 6(i)				
(i)	$[0.2 \times g \times 2.5$ or $½(0.2 + 0.8)v^2]$	M1		Finding PE loss or KE gain (system)
	$[0.2 \times g \times 2.5 = ½(0.2 + 0.8)v^2]$	M1		Using PE loss = KE gain and find v
	$[v^2 = 10]$	A1		
	$[2.5 = ½ (0 + \sqrt{10})t]$	M1		For using $s = ½(u + v)t$
	$t = 1.58$ s	A1	5	Allow $t = \frac{1}{2}\sqrt{10}$

Second Alternative Method for 6(i)				
(i)	[$T = 0.8a$ $2 - T = 0.2a$ $\rightarrow T = 1.6$ N]	M1		Apply N2 to A and B and solve for T
	[$T \times 2.5 = ½ (0.8) v^2$]	M1		Use WD by T = KE gain by A, find v
	[$v^2 = 10$]	A1		
	[$2.5 = ½ (0 + \sqrt{10})t$]	M1		Using $s = ½(u + v)t$
	$t = 1.58$ s	A1	5	Allow $t = \frac{1}{2}\sqrt{10}$
(ii)	$N = 8$ and $F = 0.1 \times N = 0.8$	B1		
	$T - 0.8 = 0.8a$ and $2 - T = 0.2a$ or $0.2g - 0.8 = (0.2 + 0.8)a$	M1		For applying N2 to both particles or to the system and solving for a
	$a = 1.2$	A1		
	$v^2 = 0 + 2 \times 1.2 \times 2.5$	M1		For using $v^2 = u^2 + 2as$
	$v = \sqrt{6} = 2.45$ ms^{-1}	A1	5	
First Alternative Method for 6(ii)				
(ii)	$N = 8$ and $F = 0.1 \times N = 0.8$	B1		
	[$0.2 \times g \times 2.5 = ½ (0.8 + 0.2) v^2 + 0.8 \times 2.5$]	M1		Apply work/energy to the system as PE loss = KE gain + WD against resistance
		A1		Correct Work/Energy equation
		M1		For solving for v
	$v = \sqrt{6} = 2.45$ ms^{-1}	A1	5	
Second Alternative Method for 6(ii)				
(ii)	$N = 8$ and $F = 0.1 \times N = 0.8$	B1		
	$T - 0.8 = 0.8a$ and $2 - T = 0.2a$	M1		Use N2 for A and B and solve for T
	$T = 1.76$ N	A1		
	[$T \times 2.5 = 0.8 \times 2.5 + ½ (0.8) v^2$]	M1		Apply Work/Energy equation to A
	$v = \sqrt{6} = 2.45$ ms^{-1}	A1	5	

7 (i)	$k = 40$	B1	1	
(ii)	Correct for $0 \leqslant t \leqslant 4$	B1√		Quadratic curve with minimum at $t = 1$ approximately, $v = 0$ at $t = 2$ and $v = k$ at $t = 4$. ft on k
	Correct for $4 \leqslant t \leqslant 14$	B1√		Horizontal line at $v = k$. ft on k
	Correct $14 \leqslant t \leqslant 20$	B1√	3	Line with negative gradient from $(14, k)$ to $(20, 28)$. ft on k
(iii)	For $0 \leqslant t \leqslant 4$ $\quad a = 10t - 10$	M1		Attempting to differentiate to find a
	$1 < t \leqslant 4$	A1	2	
(iv)	$\int(5t^2 - 10t)dt = \frac{5}{3}t^3 - 5t^2$	M1		For attempting to integrate the given quadratic expression and attempting to apply limits over the interval $t = 0$ to $t = 4$
	$A = \left[\frac{5}{3}t^3 - 5t^2\right]_0^2 = \left(\frac{5}{3}2^3 - 5\times 2^2\right) - \left(\frac{5}{3}0^3 - 5\times 0^2\right)$			Use of limits to obtain A, the integral from $t = 0$ to $t = 2$ and B, the integral from $t = 2$ to $t = 4$. Full evaluation of A not necessary at this stage $\left[A = -\frac{20}{3}\right]$
	$B = \left[\frac{5}{3}t^3 - 5t^2\right]_2^4 = \left(\frac{5}{3}4^3 - 5\times 4^2\right) - \left(\frac{5}{3}2^3 - 5\times 2^2\right)$	A1		Full evaluation of B not necessary at this stage $\left[B = \frac{100}{3}\right]$
	$C = (40 \times 10) + 0.5 \times (40 + 28) \times 6$	B1√		For finding the distance travelled in the interval $t = 4$ to $t = 20$ using area properties or integration. ft on k
	$-A + B + C = [20/3 + 100/3 + 400 + 204]$	M1		For attempting to evaluate the total distance travelled by P in the interval $t = 0$ to $t = 20$. The distance travelled in the first 4 seconds must have been found using integration methods.
	Total distance travelled = 644 m	A1	5	